

Exercise 8

Find an equation of the tangent line to the curve at the given point.

$$y = \frac{2x+1}{x+2}, \quad (1, 1)$$

Solution

Start by finding the slope of the tangent line to the curve at $x = 1$.

$$\begin{aligned} m &= \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{\frac{2x+1}{x+2} - \frac{2(1)+1}{(1)+2}}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{\frac{2x+1}{x+2} - 1}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{\frac{2x+1}{x+2}(x+2) - 1(x+2)}{(x-1)(x+2)} \\ &= \lim_{x \rightarrow 1} \frac{(2x+1) - (x+2)}{(x-1)(x+2)} \\ &= \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(x+2)} \\ &= \lim_{x \rightarrow 1} \frac{1}{x+2} \\ &= \frac{1}{(1)+2} \\ &= \frac{1}{3} \end{aligned}$$

The general equation of a line is

$$y = mx + b.$$

Here the slope is $m = 1/3$.

$$y = \frac{1}{3}x + b$$

Use the fact that the line passes through $(1, 1)$ to determine b .

$$1 = \frac{1}{3}(1) + b$$

$$1 = \frac{1}{3} + b$$

$$b = \frac{2}{3}$$

Therefore,

$$y = \frac{1}{3}x + \frac{2}{3}.$$

Below is a plot of the curve and the tangent line at $x = 1$.

